

via a scalar hyperfine coupling of magnitude A . The spin Hamiltonian describing I in a magnetic field (H_0) can then be written

$$\mathcal{H} = \mathcal{H}_{\text{EZ}} + \mathcal{H}_{\text{NZ}} + \mathcal{H}_{\text{S}\cdot\text{S}} + \mathcal{H}_{\text{I}\cdot\text{S}} + \mathcal{H}_{\text{S}\cdot\text{L}} + \mathcal{H}_{\text{D}} \quad (2)$$

where the first two parts are the electron and nuclear Zeeman energies; the third and fourth parts describe the scalar electron exchange and scalar hyperfine couplings; and the last two terms represent spin-orbit coupling and all dipolar interactions, respectively. We eliminate \mathcal{H}_{NZ} as unimportant and neglect \mathcal{H}_{D} .⁶ Furthermore, recognizing that $\mathcal{H}_{\text{S}\cdot\text{L}}$, in a freely tumbling radical in solution, shifts the g factor from the value of the free electron, we can combine \mathcal{H}_{EZ} and $\mathcal{H}_{\text{L}\cdot\text{S}}$ by introducing g_1 and g_2 as the isotropic g factors of components 1 and 2 of I, respectively.⁷ Expressing all energies in angular frequency units, we write the Hamiltonian as

$$\mathcal{H} = \beta H_0(g_1 S_1 + g_2 S_2) - J(1/2 + 2S_1 \cdot S_2) + AI \cdot S_1 \quad (3)$$

where β is the Bohr magneton, S_1 , S_2 , and I are the electron and nuclear spin operators, and J is the scalar electron exchange coupling constant.⁸ For small values of J we write the wave function of I as a mixture of the unperturbed electron singlet function, $S = 2^{-1/2}(\alpha\beta - \beta\alpha)$, and the triplet component, $T_0 = 2^{-1/2}(\alpha\beta + \beta\alpha)$. Mixing with $|1,1\rangle$ and $|1,\bar{1}\rangle$ is considered unimportant because of the large energy gap.^{1,3} Using the method of variation of constants to describe the time dependence we obtain eq 4, the wave function for the two nuclear spin states, α_N and β_N . From the

$$\Psi^+(t) = [C_S^+(t)S + C_{T_0^+}(t)T_0]\alpha_N \quad (4)$$

$$\Psi^-(t) = [C_S^-(t)S + C_{T_0^-}(t)T_0]\beta_N$$

time-dependent Schrodinger equation and (3), we get

$$i \frac{\partial C_S^\pm}{\partial t} = C_S^\pm J + C_{T_0^\pm}(1/2\beta H_0 \Delta g \pm 1/4A) \quad (5)$$

$$i \frac{\partial C_{T_0^\pm}}{\partial t} = C_S^\pm(1/2\beta H_0 \Delta g \pm 1/4A) - C_{T_0^\pm} J \quad (6)$$

where $\Delta g = g_1 - g_2$.

Integration depends on the state of I at the instant of its formation ($t = 0$). If we consider the case of a triplet precursor ($m = 3$), then $C_{T_0^\pm}(0) = 1$ and $C_S^\pm(0) = 0$.

$$C_S(t)^\pm = -i \frac{1/2\beta H_0 \Delta g \pm 1/4A}{D^\pm} \sin D^\pm t$$

$$C_{T_0^\pm}(t) = \cos D^\pm t - i \frac{J}{D^\pm} \sin D^\pm t$$

where

$$D^\pm = [(1/2\beta H_0 \Delta g \pm 1/4A)^2 + J^2]^{1/2}$$

Since product formation should depend on the degree of singlet character in I it should be proportional to

$$[C_S^\pm(t)]^2 = \frac{(1/2\beta H_0 \Delta g \pm 1/4A)^2}{D^{\pm 2}} \sin^2 D^\pm t \quad (7)$$

(6) The justification for this simplification will be presented in a future publication.

(7) The g shift arises from an interplay of spin-orbit ($S \cdot L$) and orbit-Zeeman interaction ($H \cdot L$). Cf. C. Slichter, "Principles of Magnetic Resonance," Harper and Row, New York, N. Y., 1963, p 179.

(8) The exchange coupling is written here in different form from that used in ref 1 and differs from the previous treatment by a factor of -2 . We have adopted this form to achieve uniformity with that used in ref 3.

eq 7. Examination of (7) shows that products with different nuclear spin states will be formed with different rates, attributable to the dependence of the mixing coefficient on the nuclear spin state, $1/2\beta H_0 \Delta g + 1/4A$ for α_N and $1/2\beta H_0 \Delta g - 1/4A$ for β_N . It also shows that if $\Delta g = 0$ both spin states build up in the product with equal rates giving no polarization. The sign of the polarization depends on the relative signs of A and Δg and, therefore, the same mechanism can account for both signal enhancement and emission. As pointed out before,^{1,3} the magnitude of the polarization depends also on the mean lifetime of I. Averaging over the lifetime, τ , gives the rates of population increase of the two nuclear spin states (w^+ for α_N and w^- for β_N) in the cage product (eq 8) where k_{SE} is the specific rate

$$w^\pm = k_{\text{SE}} \frac{2(1/2\beta H_0 \Delta g \pm 1/4A)^2 \tau^2}{1 + 4D^{\pm 2} \tau^2} \quad (8)$$

constant for cage product formation from the pure singlet state of the radical pair. If I is derived from a singlet precursor ($m = 1$), product formation should be proportional to eq 9, which leads to the opposite

$$w^\pm = k_{\text{SE}} \left[1 - \frac{2(1/2\beta H_0 \Delta g \pm 1/4A)^2 \tau^2}{1 + 4D^{\pm 2} \tau^2} \right] \quad (9)$$

polarization as observed from a triplet precursor.

We believe that this mechanism is superior to the Fischer mechanism, which is based on cross-relaxations in free radicals,⁹ because (i) it allows both entropy and energy polarization to be explained by the same phenomenon, namely singlet-triplet mixing in radical pairs by hyperfine interaction and different g shifts in the components of the radical pair; (ii) it provides for different and predictable signs of polarization with no need for different interaction mechanisms between electron and nuclear spin; (iii) it puts no Overhauser limit on the magnitude of the polarization (assuming for a hypothetical reaction proceeding via a triplet precursor with $\Delta g = 10^{-3}$, $A = 2.5 \times 10^8$, $J \sim 10^3$ radians/sec, $\tau = 10^{-9}$ sec, and 98% product formation via nuclear spin state independent mechanisms, one obtains an enhancement factor of 4×10^3 at an nmr observation frequency of 60 MHz at 300°K); (iv) the polarization should be a function of H_0 ; (v) this mechanism puts no limit on the lifetime of the free radicals because polarization occurs only in radical pairs.

The accompanying communications describe experimental results which are readily explained by this mechanism but are difficult to understand with the previous model.

Acknowledgment. We are indebted to R. Kaptein, L. J. Oosterhoff, M. Lehnig, and H. Fischer for communicating their results before publication.

(9) H. Fischer and J. Bargon, *Accounts Chem. Res.*, **2**, 110 (1969), and references therein.

(10) Cf. M. Lehnig and H. Fischer, *Z. Naturforsch.*, in press; H. R. Ward, R. G. Lawler, H. Y. Loken, and R. A. Cooper, *J. Amer. Chem. Soc.*, **91**, 4929 (1969). Our model is not valid at very low field because it neglects mixing with $|1,1\rangle$ and $|1,\bar{1}\rangle$ which becomes important in the low field region.

G. L. Closs, A. D. Trifunac

Department of Chemistry, The University of Chicago
Chicago, Illinois 60637

Received January 13, 1970